

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel International GCSE

Time 2 hours

Paper
reference**4MA1/2HR**

Mathematics A

PAPER 2HR

Higher Tier



You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain **NO** credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

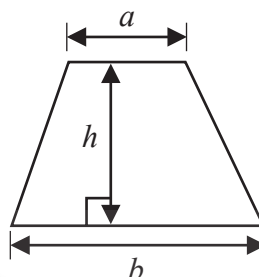
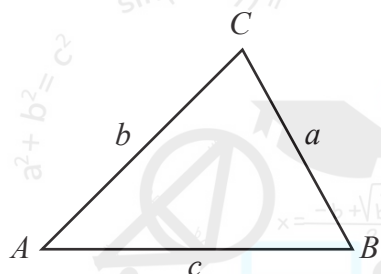
Sum to n terms, $S_n = \frac{n}{2} [2a + (n-1)d]$

The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium = $\frac{1}{2}(a+b)h$

**Trigonometry****In any triangle ABC**

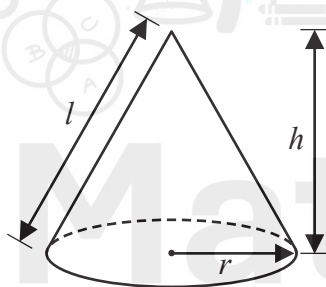
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

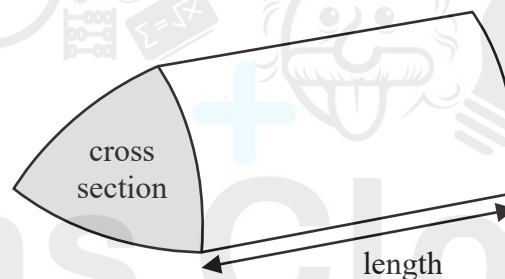
Area of triangle = $\frac{1}{2}ab \sin C$

Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

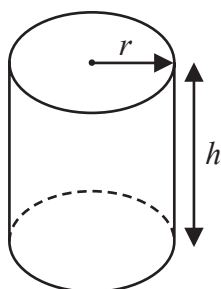
**Volume of prism**

= area of cross section \times length



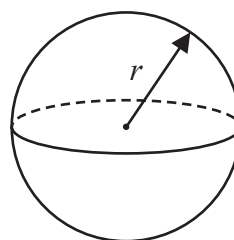
Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi r h$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



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Answer ALL TWENTY SIX questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Show that $4\frac{2}{3} \div 1\frac{5}{6} = 2\frac{6}{11}$

let $1 = \frac{3}{3}$, then $4 = \frac{3}{3} \times 4 = \frac{12}{3}$
 $\frac{12}{3} + \frac{2}{3} = \frac{14}{3}$

let $1 = \frac{6}{6}$, then $\frac{6}{6} + \frac{5}{6} = \frac{11}{6}$

So $4\frac{2}{3} \div 1\frac{5}{6}$ can be expressed as:

$$\frac{14}{3} \div \frac{11}{6}$$

Using keep, flip, change method:

Keep: $\frac{14}{3}$, Flip: $\frac{11}{6}$ to $\frac{6}{11}$, change: \div to \times

$$\frac{14}{3} \times \frac{6}{11} = \frac{84}{33}$$

$\frac{84}{33}$ can be simplified by $\div 3$

$$\frac{84}{33} \div 3 = \frac{28}{11}$$

let $1 = \frac{11}{11}$ $28 - 22 = 6$

$$\frac{28}{11} = 2\frac{6}{11}$$

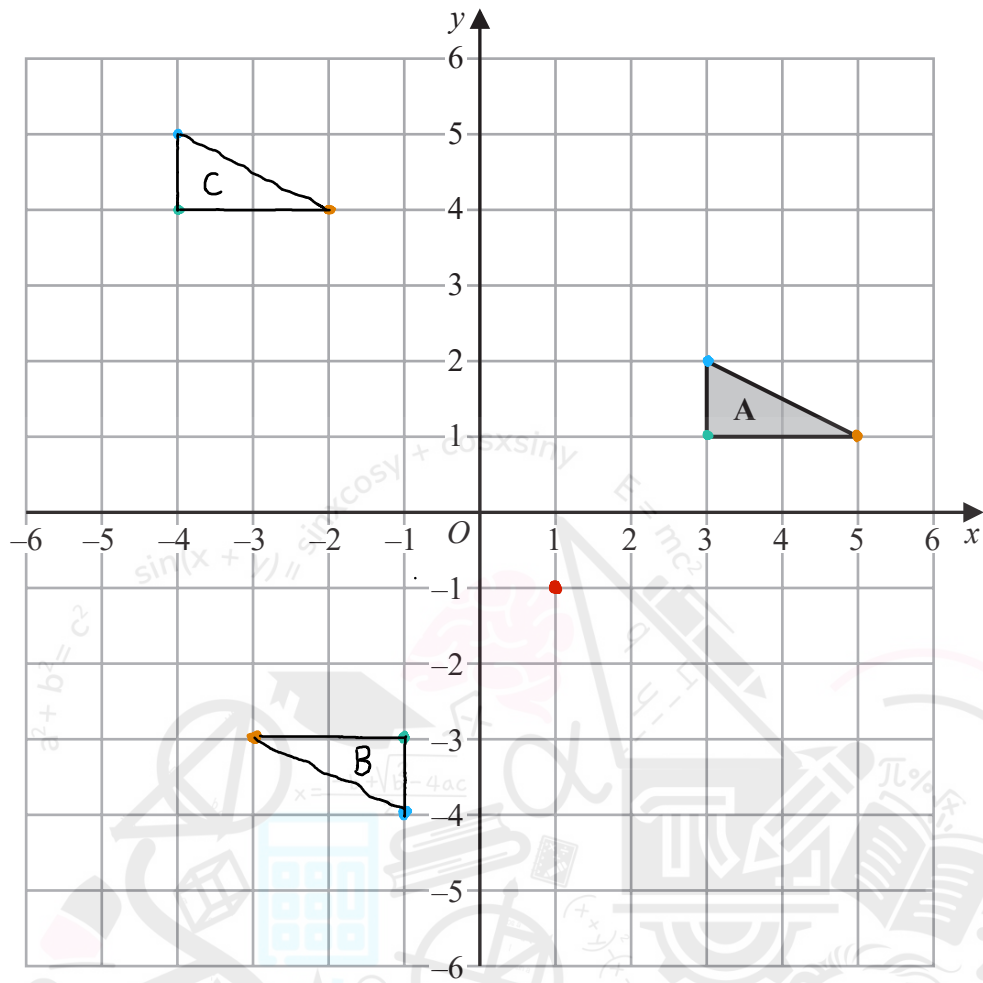
as $28 \div 11 = 2.54\dots$,
so we put 2 here

Therefore, $4\frac{2}{3} \div 1\frac{5}{6} = 2\frac{6}{11}$

(Total for Question 1 is 3 marks)



2



(a) On the grid, rotate triangle A 180° about $(1, -1)$
 Label the new triangle B (2)

(b) On the grid, translate triangle A by the vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$
 Label the new triangle C (1)

(Total for Question 2 is 3 marks)



3 $-8 < 2y \leq 2$

y is an integer.

(a) Find all the possible values of y

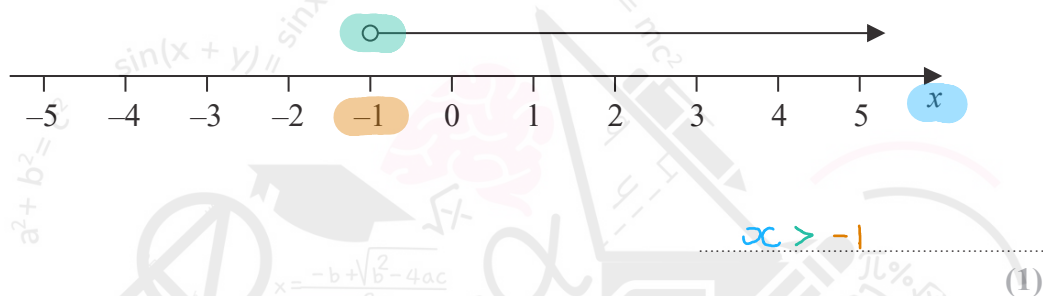
$$-8 < 2y \leq 2 \quad (\div 2)$$

$$-4 < y \leq 1$$

So $y = -3, -2, -1, 0, 1$

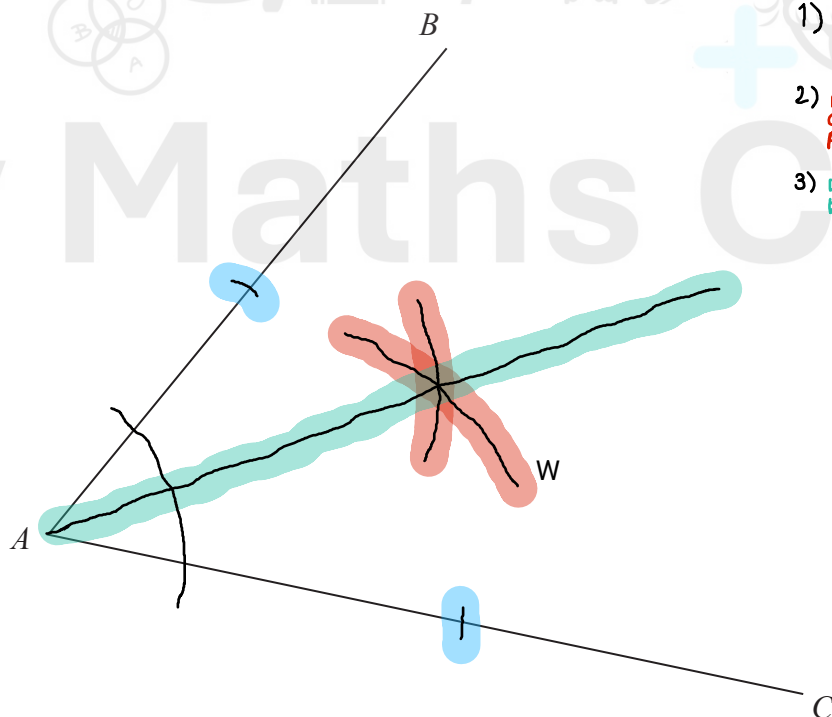
$-3, -2, -1, 0, 1$ (2)

(b) Write down the inequality shown on the number line.



(Total for Question 3 is 3 marks)

4 Using ruler and compasses only, construct the bisector of angle BAC . You must show all your construction lines.



Method:

- 1) Find the midpoint of A and B.
- 2) mark the bisector from the compass being on the midpoints of A and B.
- 3) Draw the angle line in from these points.

(Total for Question 4 is 2 marks)



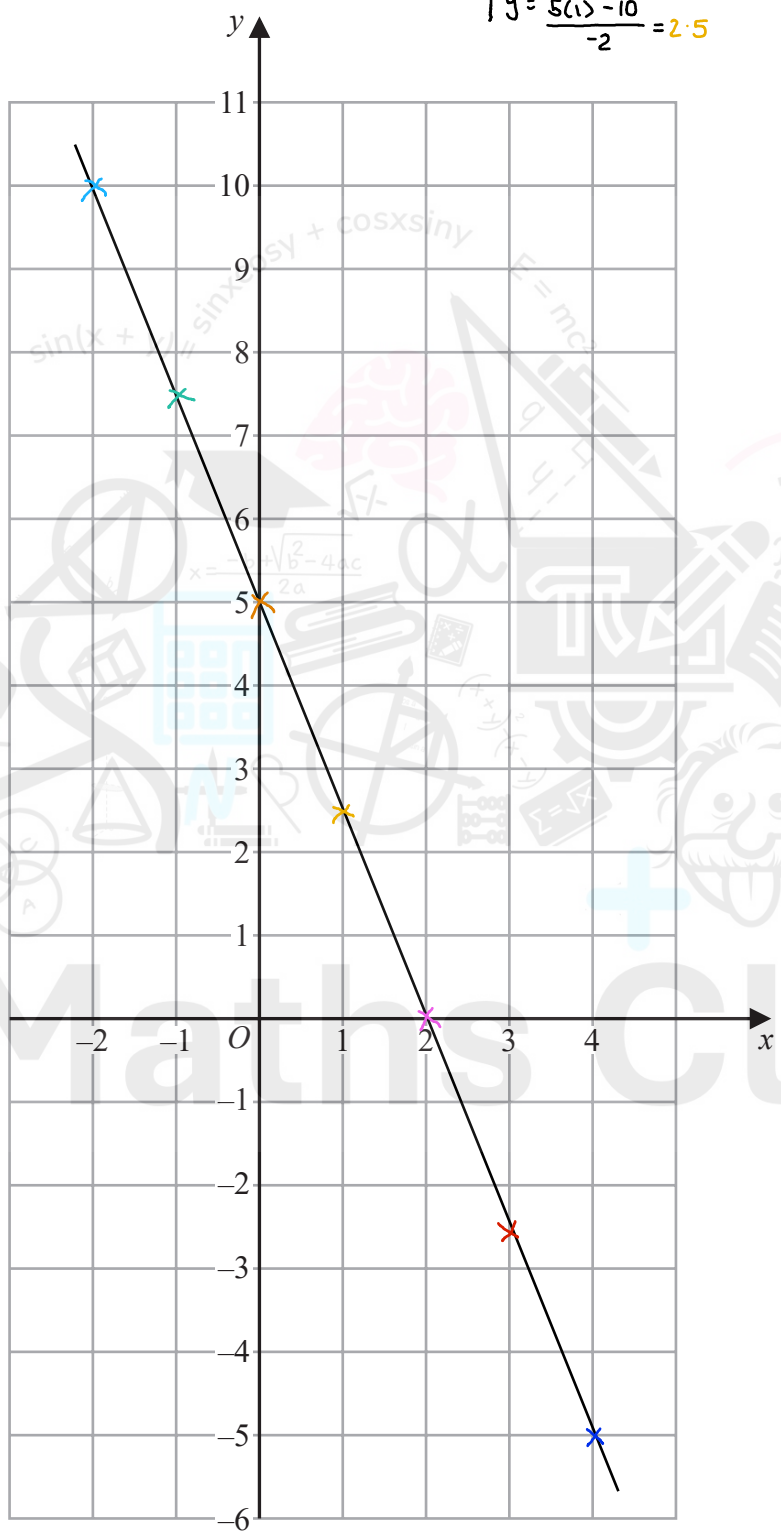
rearrange to get y:
 $5x + 2y = 10$
 $5x = 10 - 2y$
 $5x - 10 = -2y$
 $y = \frac{5x - 10}{2}$

5 On the grid, draw the graph of $5x + 2y = 10$ for values of x from -2 to 4

x	-2	-1	0	1	2	3	4
y	10	7.5	5	2.5	0	-2.5	-5

$y = \frac{5(-2) - 10}{2} = 10$
 $y = \frac{5(-1) - 10}{2} = 7.5$
 $y = \frac{5(0) - 10}{2} = 5$
 $y = \frac{5(1) - 10}{2} = 2.5$

$y = \frac{5(2) - 10}{2} = 0$
 $y = \frac{5(3) - 10}{2} = -2.5$
 $y = \frac{5(4) - 10}{2} = -5$



(Total for Question 5 is 3 marks)

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- 6 In a bag, there are only red counters, blue counters, green counters and yellow counters.

The total number of counters in the bag is 80

In the bag

the number of red counters is $x + 7$

the number of blue counters is $x - 11$

the number of green counters is $3x$

Jude takes at random a counter from the bag.

The probability that he takes a red counter is $\frac{1}{4}$

Work out the probability that Jude takes a yellow counter.

If the probability of picking a red counter is $\frac{1}{4}$ and the total number of counters is 80, then the number of red counters is:

$$80 \times \frac{1}{4} = 20 \text{ red counters}$$

$$80, x + 7 = 20$$

$$x = 13$$

$$\text{Number of blue counters: } x - 11$$

$$13 - 11 = 2 \text{ blue counters}$$

$$\text{Number of green counters: } 3x$$

$$3(13) = 39 \text{ green counters}$$

$$\text{Number of yellow counters: } 80 - (20 + 2 + 39)$$

$$80 - 61 = 19 \text{ yellow counters}$$

$$\text{probability of picking a yellow counter: } \frac{19}{80}$$

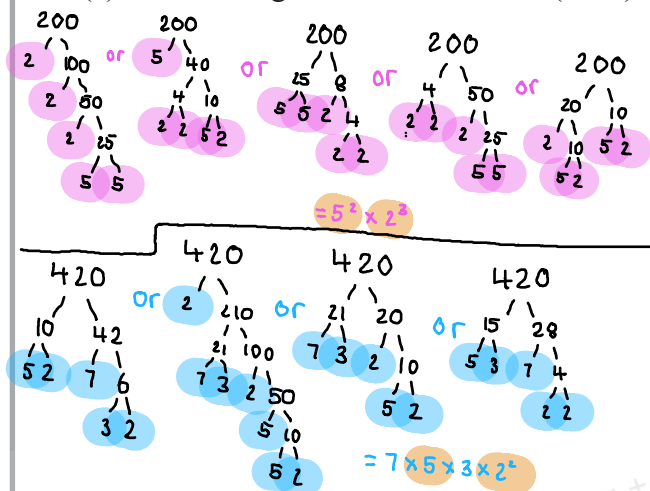
$$\frac{19}{80}$$

(Total for Question 6 is 4 marks)



Or using the ladder method:

7 (a) Find the highest common factor (HCF) of 200 and 420



Write 200 with a L:
 $2 \mid 200$
 $2 \mid 100$
 $5 \mid 50$
 $5 \mid 10$
 $2 \mid 2$

Find a factor and write it on the outside of the ladder:
 $2 \mid 200$

Divide 200 by the factor and write the result underneath:
 $2 \mid 200$
 100

Draw another L and repeat the process until there is no number 200 can be divided by:
 $2 \mid 200$
 $2 \mid 100$
 $5 \mid 50$
 $5 \mid 10$
 $2 \mid 2$

The remaining outside numbers are the prime factors.
 Prime factors of 200: $5^2 \times 2^3$

Repeat the process for 420:
 $2 \mid 420$
 $2 \mid 210$
 $5 \mid 105$
 $7 \mid 21$
 $3 \mid 3$

Prime factors of 420: $7 \times 5 \times 3 \times 2^2$
 Prime factors of 300: $5^2 \times 2^3$

Both 420 and 300 share a 5 and 2^2 in their prime factors
 HCF of 420 and 300: $5 \times 2^2 = 20$

..... 20
 (2)

420 and 300 both share 5 and 2^2 in their prime factors.
 HCF of 420 and 300: $5 \times 2^2 = 20$

$A = 2^3 \times 3 \times 5 \times 7^2$
 $B = 2 \times 3^2 \times 7$
 $C = 3 \times 5^2 \times 11$

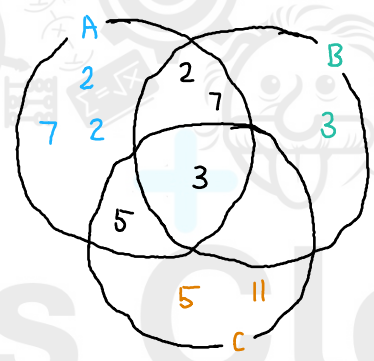
(b) Find the lowest common multiple (LCM) of A, B and C
 Write your answer as a product of powers of prime factors.

$A = 2^3 \times 3 \times 5 \times 7^2$
 $B = 2 \times 3^2 \times 7$
 $C = 3 \times 5^2 \times 11$

LCM = $2^3 \times 3^2 \times 5^2 \times 7^2 \times 11$

Note: We want the lowest power of each value to calculate LCM, which is in contrast to HCF (highest common factor).

Or use a venn diagram:



$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 11$
 LCM = $2^3 \times 3^2 \times 5^2 \times 7^2 \times 11$

..... $2^3 \times 3^2 \times 5^2 \times 7^2 \times 11$
 (2)

(Total for Question 7 is 4 marks)



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8 60 students sat a Mathematics exam.

The mean mark for the 32 students in Class A was 55

The mean mark for the 28 students in Class B was 52

Find the mean mark for all 60 students.

Class A: Total marks for
the class $\div 32 = 55$

Total marks for class A: $55 \times 32 = 1760$

Class B: Total marks for
the class $\div 28 = 52$

Total marks for class B: $52 \times 28 = 1456$

Total marks for
both classes: $1760 + 1456 = 3216$

mean mark for
all 60 students: $\frac{3216}{60} = 53.6$

53.6

(Total for Question 8 is 3 marks)

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P 7 2 4 4 4 A 0 9 3 2

- 9 Teresa invests \$2000 for 3 years in a savings account. She gets 4% each year compound interest.

- (a) How much money will Teresa have in her savings account at the end of 3 years?
Give your answer correct to the nearest dollar.

4% Compound interest can be expressed as: 1.04

$$= 2000 \times 1.04^3 = 2249.728$$

as over 3 years

To the nearest dollar: \$2250

\$ 2250
(3)

- Sam invested \$T
The value of his investment decreased by 9% each year.

At the end of the first year, the value of Sam's investment was \$1365

- (b) Work out the value of T

9% decrease can be expressed as: 0.91

$$T \times 0.91 = 1365$$

$$T = 1365 \div 0.91$$

$$T = 1500$$

1500
(3)

(Total for Question 9 is 6 marks)

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10 The diagram shows two solids, A and B, made from two different metals.

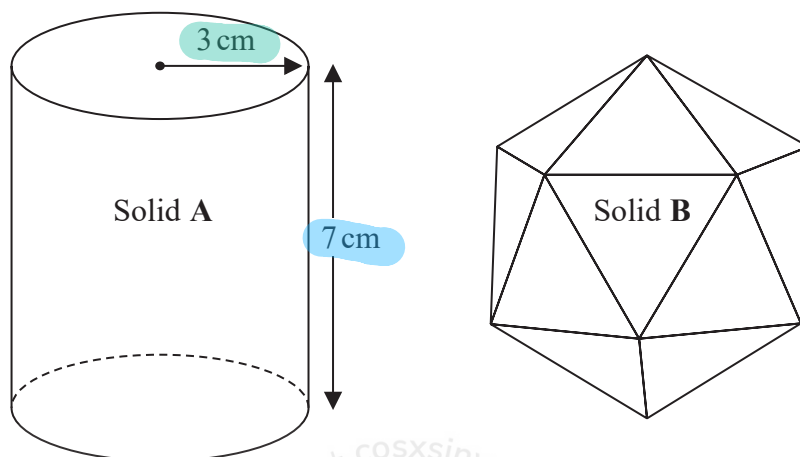


Diagram NOT accurately drawn

Solid A is in the shape of a cylinder with radius 3 cm and height 7 cm

Solid A has a mass of 2000 g

Solid B has a mass of 3375 g

Solid B has a volume of 450 cm³

All of the metal from Solid A and Solid B is melted down to make a uniform Solid C

Given that there is no change to mass or volume during this process

work out the density of Solid C

Give your answer correct to one decimal place.

Find the volume of solid A

$$\text{Volume} = \frac{\text{face surface area}}{\text{area}} \times 7$$

$$\text{face surface area} = \pi r^2$$

$$r (\text{radius}) = 3$$

$$\begin{aligned} \text{face surface area} &= \pi 3^2 \\ &= 9\pi \end{aligned}$$

(keep the answer in terms of π to retain accuracy in subsequent calculations)

$$\begin{aligned} \text{Volume} &= 9\pi \times 7 \\ &= 63\pi \end{aligned}$$

Find the density of the solids:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{Density of solid A} = \frac{2000}{63\pi}$$

$$\text{Density of solid B} = \frac{3375}{450}$$

Keep as fractions to retain accuracy in further calculations

$$\text{Density of Solid C} = \frac{2000 + 3375}{63\pi + 450} = 8.295\dots$$

$$\text{rounded to 1 decimal place} : 8.3 \text{ g/cm}^3$$

8.3 g/cm³

(Total for Question 10 is 3 marks)

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11

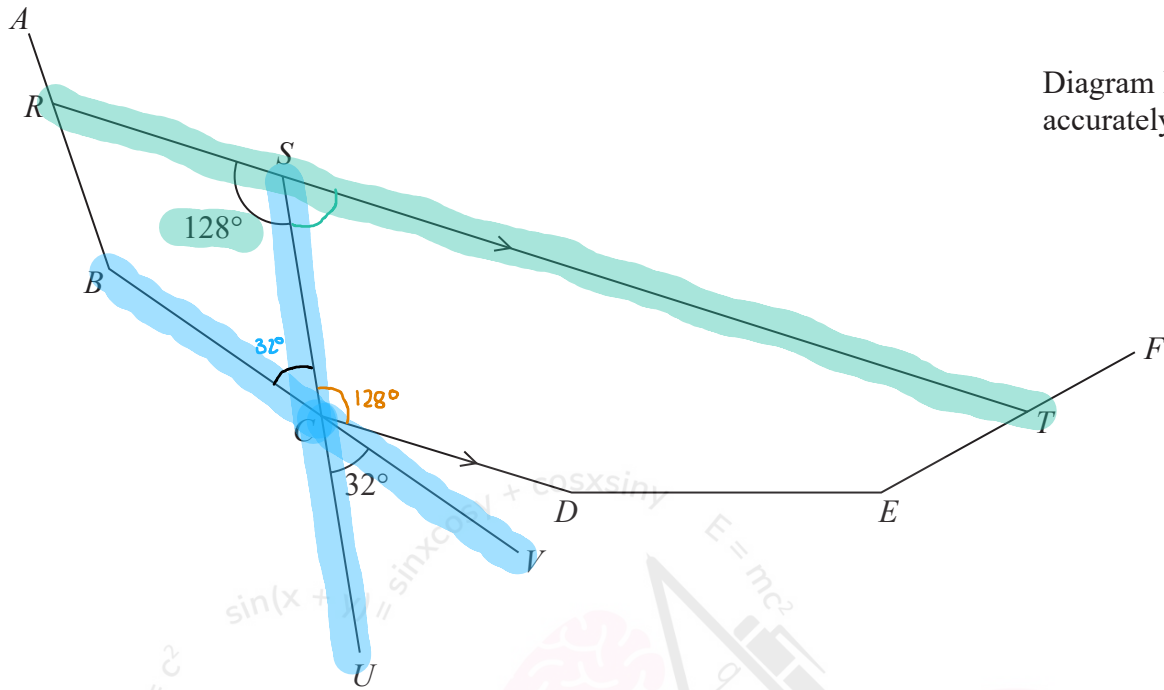


Diagram NOT accurately drawn

AB, BC, CD, DE and EF are five sides of a regular polygon.

RST, SCU and BCV are straight lines.

RST is parallel to CD

Angle $RSC = 128^\circ$

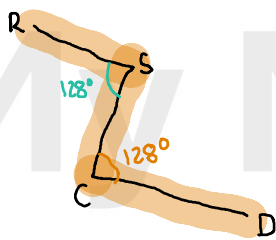
Angle $UCV = 32^\circ$

Work out how many sides the polygon has.

Show your working clearly.

Recognise that opposite angles are equal, so $\hat{BCS} = 32^\circ$

Angles on a straight line = 180° . So, $\hat{SCD} = 180 - 128 = 52^\circ$



'z' angles are equal, so $\hat{SCD} = 128^\circ$

Size of one interior angle: $\hat{BCS} + \hat{SCD}$
 $= 32^\circ + 128^\circ = 160^\circ$

Interior angle formula = $\frac{180}{n-2}$ (where n = number of sides)

Plugging in known values: $160 = \frac{180}{n-2}$

Rearranging: $160(n-2) = 180$

Expanding: $160n - 320 = 180$

$$160n = 500$$

$$n = \frac{500}{160}$$

$$n = 18$$

(Total for Question 11 is 4 marks)

18

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12 (a) Simplify $\frac{2}{y^0}$

Recognise that anything to the power of 0 is one.

$$y^0 = 1$$

Becomes: $\frac{2}{1} = 2$

$$\frac{2}{1} \quad (1)$$

(b) Simplify fully $(16a^4)^{\frac{3}{4}}$

Split up into: $16^{\frac{3}{4}} (a^4)^{\frac{3}{4}}$

$$16^{\frac{3}{4}} = 8$$

Becomes: $8(a^4)^{\frac{3}{4}}$

$$= (8a^4)^{\frac{3}{4}}$$

Apply indices rules $(k^a)^b = k^{a \times b}$

$$(8a^4)^{\frac{3}{4}} = 8^{4 \times \frac{3}{4}} = 8a^3$$

$$8a^3 \quad (2)$$

(c) Expand and simplify $5x(3x+4)(2x-1)$

Expand the brackets:

$$\begin{array}{r|l} & 3x + 4 \\ 2x & 6x^2 + 8x \\ -1 & -3x - 4 \end{array}$$

$$5x(6x^2 + 5x - 4)$$

$$\begin{array}{r|l} & 6x^2 + 5x - 4 \\ 5x & 30x^3 + 25x^2 - 20x \end{array}$$

$$= 30x^3 + 25x^2 + 20x \quad (-20x)$$

$$30x^3 + 25x^2 + 20 \quad (-20)$$

$$30x^3 + 25x^2 - 20$$

Note: $30x^3 + 25x^2 - 20x$ will gain full marks, however correct further simplifications will not be penalised.

$$30x^3 + 25x^2 - 20x \quad (3)$$

(Total for Question 12 is 6 marks)

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13 A rectangle has length L and width W

L is increased by 20%

W is decreased by 35%

Calculate the percentage reduction in the area of the rectangle.

Length is increased by 20%, so can be expressed as: $1.2L$ ($1+0.2=1.2$)
 Width is decreased by 35%, so can be expressed as: $0.65W$ ($1-0.35=0.65$)

$$\begin{aligned} \text{Area} &= \text{Length} \times \text{width} \\ &= 1.2L \times 0.65W \\ &= 0.78LW \end{aligned}$$

percentage reduction: $(1 - 0.78) \times 100$
 $= 22\%$

$\times 100$ as we want the answer in the form of a percentage

22%

(Total for Question 13 is 3 marks)

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14 A , B and C are points on a circle, centre O

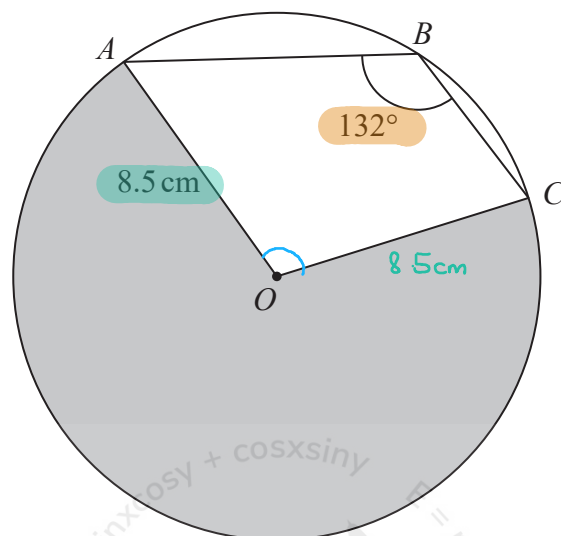


Diagram NOT
accurately drawn

The radius of the circle is 8.5 cm
Angle $ABC = 132^\circ$

Work out the perimeter of the shaded sector AOC
Give your answer correct to 3 significant figures.

$$\text{Angle } AOC = 2 \times 132$$

$$AOC = 264^\circ$$

$$\text{perimeter} = \text{AC arc length} + (\text{radius} \times 2)$$

$$= \frac{\theta}{360} \times 2\pi r + (\text{radius} \times 2)$$

$$= \frac{264}{360} \times 2 \times \pi \times 8.5 + (8.5 \times 2)$$

$$= 56.1651\dots$$

rounded to
3 significant
figures : 56.2

..... 56.2 cm

(Total for Question 14 is 3 marks)



15 Here are the numbers of aces that Rutger served in each of 11 tennis matches.

1 1 2 4 6 8 8 9 11 12 15

(a) Find the interquartile range of the numbers of aces.

Show your working clearly.

$$\text{IQR} = 75\% \text{ value} - 25\% \text{ value}$$

$$\parallel \times 0.25 = 2.75, \text{ so take the } 3^{\text{rd}} \text{ value (2)}$$

$$\parallel \times 0.75 = 8.25, \text{ so take the } 9^{\text{th}} \text{ value (11)}$$

$$\text{IQR} = 11 - 2 = 9$$

9

(2)

Kim also plays in 11 tennis matches.

For Kim

the median number of aces is 11

the interquartile range of the numbers of aces is 5

(b) State, giving a reason, whether Rutger or Kim

(i) served more aces on average,

Kim, as she has a higher median or Kim as her median is 11 whereas Rutger's is 8 or Kim's median is 3 more or Kim as Rutger's median is 3 less.

(1)

(ii) was more consistent with the number of aces served.

Kim as she has a smaller IQR or Kim as her IQR is 5 whereas Rutger's is 9 or Kim's IQR is 4 less or Kim as Rutger's IQR is 4 more.

(1)

(Total for Question 15 is 4 marks)

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16 Here are two vectors.

$$\vec{BA} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

Find \vec{AC} as a column vector.

$$\begin{aligned} \vec{AC} &= \vec{BC} - \vec{BA} \\ &= \begin{pmatrix} 9 \\ 1 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 14 \\ -3 \end{pmatrix} \end{aligned}$$

$$\vec{AC} = \begin{pmatrix} 14 \\ -3 \end{pmatrix}$$

(Total for Question 16 is 2 marks)

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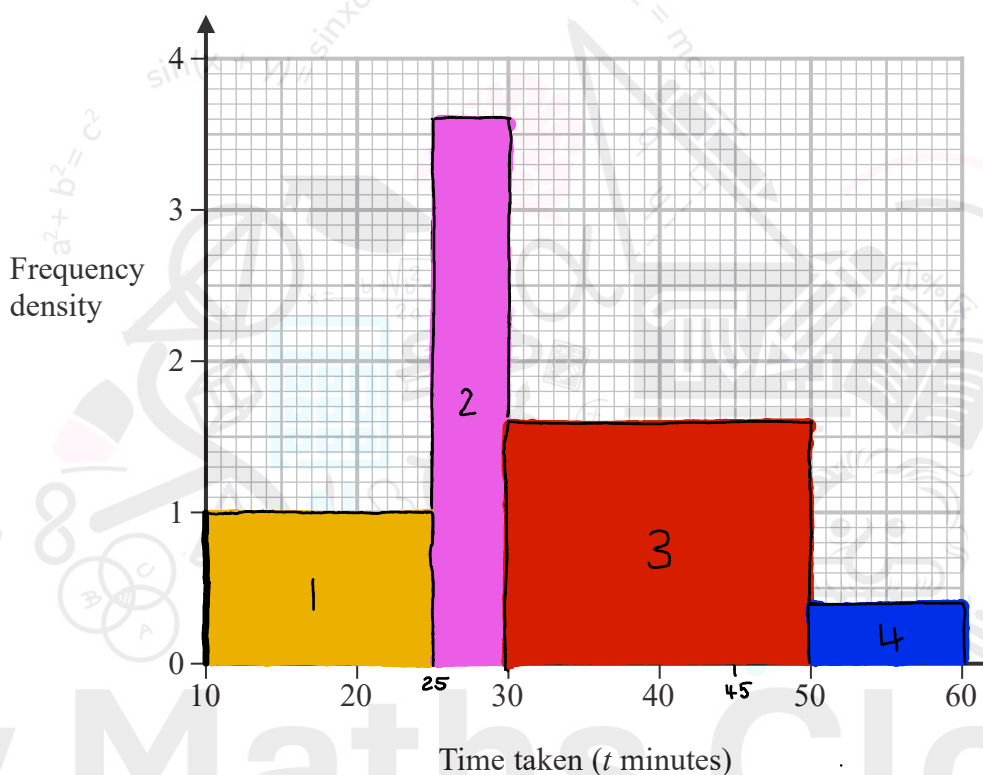
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- 17 The table gives information about the time taken by each student in Year 11 to complete a homework task.

Time taken (t minutes)	Frequency	Time taken upper bound - time taken lower bound ($=x$)	Frequency density (frequency $\div x$)
1 $10 < t \leq 25$	15	$25 - 10 = 15$	$15 \div 15 = 1$
2 $25 < t \leq 30$	18	$30 - 25 = 5$	$18 \div 5 = 3.6$
3 $30 < t \leq 50$	32	$50 - 30 = 20$	$32 \div 20 = 1.6$
4 $50 < t \leq 60$	4	$60 - 50 = 10$	$4 \div 10 = 0.4$

- (a) On the grid, draw a histogram for this information.



One of these students who took 50 minutes or less and more than 25 minutes to complete this homework task is chosen at random.

- (b) Find an estimate for the probability that this student took 45 minutes or less to complete this homework task.

50 mins or less, more than 25 mins and 45 mins or less = between 25 mins and 45 mins
 $45 - 25 = 20$
 25 - 50 mins and 25 - 45 mins = difference of 5 mins.

$$\begin{aligned} \text{Estimate} &= \frac{50}{50} - \left(\frac{5}{20} \times 32 \right) \\ &= \frac{50}{50} - \frac{8}{50} \\ &= \frac{42}{50} \end{aligned}$$

$$\begin{array}{r} 42 \\ 50 \\ \hline \end{array} \quad (2)$$

(Total for Question 17 is 5 marks)



18 A statue and a model of the statue are mathematically similar.

The statue has a total surface area of 3600 cm^2

The model has a total surface area of 625 cm^2

The volume of the model is 750 cm^3

Work out the volume of the statue.

Note: We can calculate a shared scale factor as we are told the statues are mathematically similar.

$$\text{Scale factor: } \sqrt{\frac{3600}{625}} = \frac{12}{5}$$

$$\text{Volume of statue} = 750 \times \left(\frac{12}{5}\right)^3$$

$$= 10,368$$

$$\dots\dots\dots 10,368 \dots\dots\dots \text{ cm}^3$$

(Total for Question 18 is 3 marks)

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19 Prove algebraically that, for any three consecutive even numbers,

the sum of the squares of the smallest even number and the largest even number is 8 more than twice the square of the middle even number.

we can represent an even number as $2n$

3 Consecutive even numbers: $2n, 2n+2, 2n+4$

Part 1: $(2n)^2 + (2n+4)^2$

$$= 4n^2 + (2n+4)^2$$

$$\begin{array}{r} 2n \quad 4 \\ 2n \overline{) 4n^2 + 8n} \\ \underline{4n^2 + 8n} \\ 4 \quad \underline{+ 8n + 16} \end{array}$$

$$= 4n^2 + 4n^2 + 16n + 16$$

$$= 8n^2 + 16n + 16$$

Part 2: $2(2n+2)^2$

$$\begin{array}{r} 2n \quad 2 \\ 2n \overline{) 4n^2 + 8n} \\ \underline{4n^2 + 4n} \\ 2 \quad \underline{+ 4n + 4} \end{array}$$

$$= 2(4n^2 + 8n + 4)$$

$$= 8n^2 + 16n + 8$$

Part 1 + 8 = Part 2

$$8n^2 + 16n + 8 + 8 = 8n^2 + 16n + 16$$

$$8n^2 + 16n + 16 = 8n^2 + 16n + 16$$

16 is 8 more than 8 and

$$8n^2 + 16n = 8n^2 + 16n$$

\therefore we have proved the statement

(Total for Question 19 is 3 marks)

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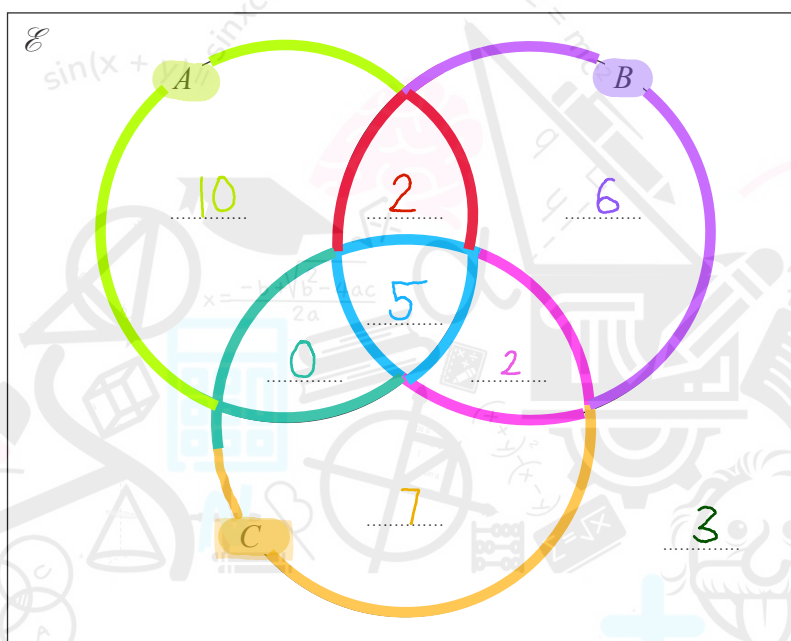
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20 A , B and C are three sets.

- $n(A \cap B \cap C) = 5$ ← everything in A , B and C
- $n(A \cap B \cap C') = 2$ ← everything shared in A and B but not in C
- $n(A \cap C) = 5$ ($5 - 5 = 0$) ← everything shared in A and C
- $n(A) = 17$ ($17 - 5 - 2 = 10$) ← we take away all known values in A from the given total
- $n([A \cup B \cup C]') = 3$ ← everything not in A , B and C
- $n(A' \cap B \cap C') = 6$ ← everything in B and not shared A and C
- $n(B \cap C) = 7$ ($5 - 7 = 2$) ← everything shared in B and C
- $n(C) = 14$ ($14 - 2 - 5 - 0 = 7$) ← we take away all known values in C from the given total.

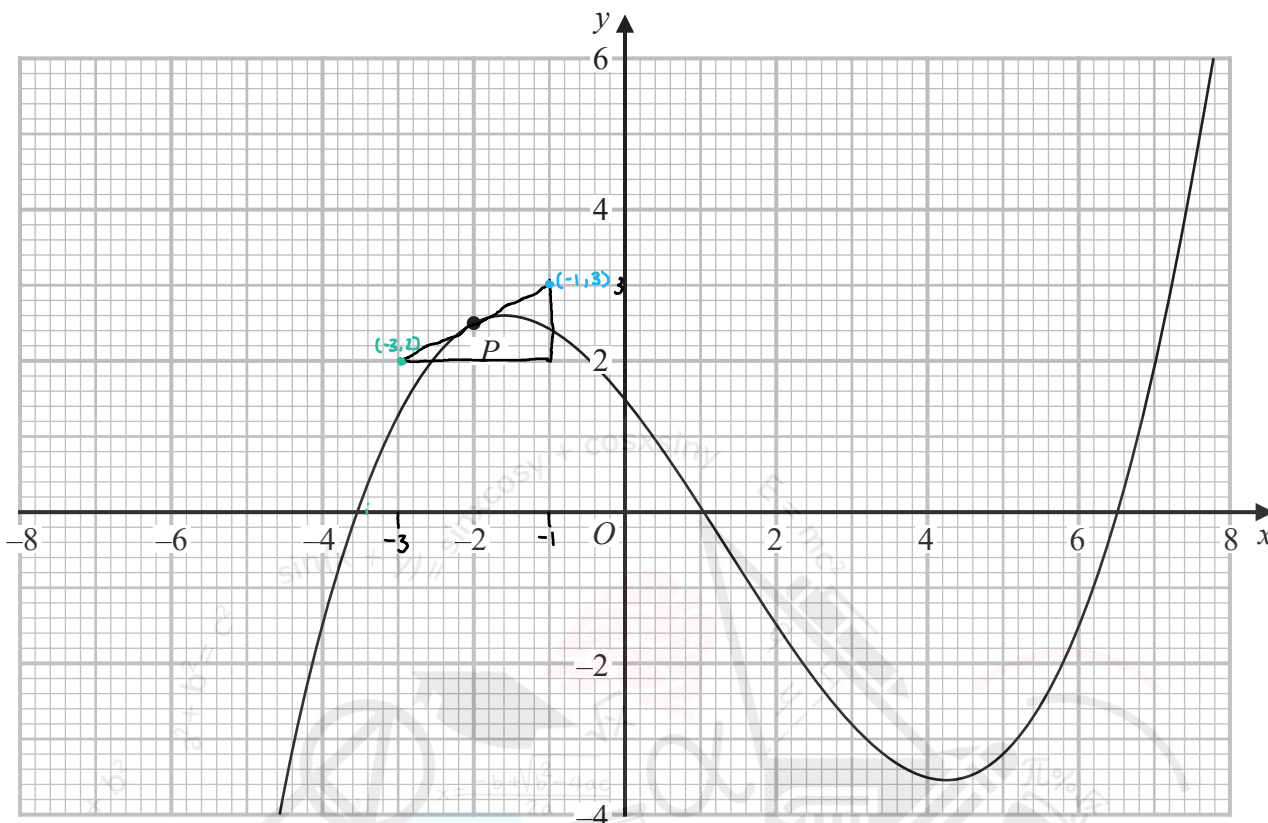
Complete the Venn diagram to show the number of elements in each region.



(Total for Question 20 is 4 marks)



21 The diagram shows the graph of $y = f(x)$



The point P has x coordinate -2

Use the graph to find an estimate for the gradient of the curve at P

From the points on the drawn tangent:

x_1 y_1
 $(-1, 3)$
 x_2 y_2
 $(-3, 2)$

Gradient: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{-3 - (-1)} = 0.5$

Note: answers in the range of 0.2 - 0.8 from correct figures from the drawn tangent.

..... 0.5

(Total for Question 21 is 3 marks)

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22 Solve the simultaneous equations

$$2y^2 + x^2 = -6x + 42$$

$$2x + y = -3$$

Show clear algebraic working.

Rearrange $2x + y = -3$ to get y on it's own

$$y = -3 - 2x$$

plug in $y = -3 - 2x$ into $2y^2 + x^2 = -6x + 42$

$$2(-3 - 2x)^2 + x^2 = -6x + 42$$

$$\begin{array}{r} -3 \quad -2x \\ -3 \left| \begin{array}{cc} 9 & 6x \\ 6x & 4x^2 \end{array} \right. \\ -2x \left| \begin{array}{cc} 6x & 4x^2 \end{array} \right. \end{array}$$

$$2(4x^2 + 12x + 9) + x^2 = -6x + 42$$

$$8x^2 + 24x + 18 + x^2 = -6x + 42$$

$$8x^2 + 24x + 18 + x^2 + 6x - 42 = 0$$

collect like terms

$$9x^2 + 30x - 24 = 0 \quad (÷3)$$

$$3x^2 + 10x - 8 = 0$$

$$a=3, b=10, c=-8$$

using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-8)}}{2(3)}$$

$$x = \frac{-10 + 14}{6}$$

$$x = \frac{-10 - 14}{6}$$

$$x = \frac{2}{3}$$

$$x = -4$$

$$y = -3 - 2x$$

$$y = -3 - 2\left(\frac{2}{3}\right)$$

$$\text{or } y = -3 - 2(-4)$$

$$y = \frac{-13}{3}$$

$$y = 5$$

$$\therefore x = \frac{2}{3}$$

$$y = \frac{-13}{3}$$

$$x = -4$$

$$y = 5$$

$$x = \frac{2}{3} \quad x = -4$$

$$y = \frac{-13}{3} \quad y = 5$$

(Total for Question 22 is 5 marks)



23 AEC and BED are chords of a circle.

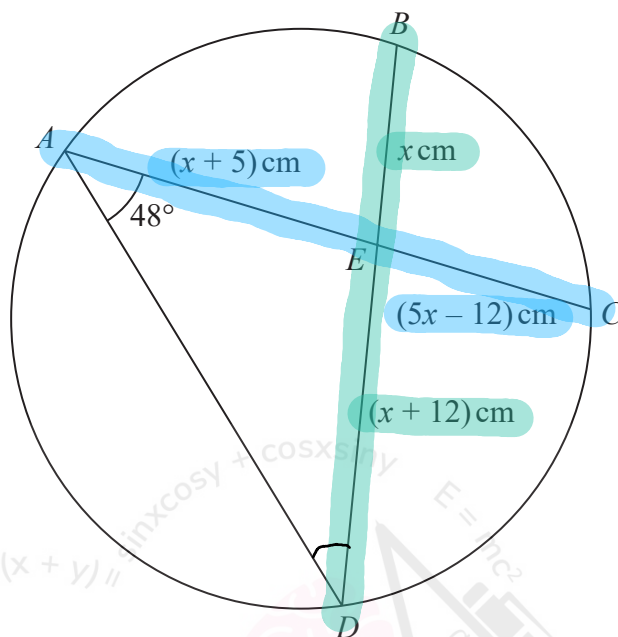


Diagram NOT accurately drawn

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$AE = (x + 5) \text{ cm}$ $BE = x \text{ cm}$ $CE = (5x - 12) \text{ cm}$ $DE = (x + 12) \text{ cm}$

Angle $DAE = 48^\circ$

Work out the size of angle ADE
Give your answer correct to one decimal place.

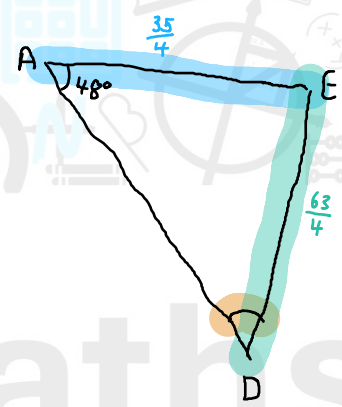
$AC = BD$
 $(x + 5)(5x - 12) = x(x + 12)$

x	5
$5x$	$5x^2 + 25x$
-12	$-12x - 60$

 $5x^2 + 13x - 60 = x^2 + 12x$
 $4x^2 + 13x - 60 = 12x$
 $4x^2 + x - 60 = 0$
 $a = 4$ $b = 1$ $c = -60$

using the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-1 \pm \sqrt{1 - 4(4)(-60)}}{2(4)}$
 $x = \frac{-1 + 31}{8}$ $x = \frac{-1 - 31}{8}$
 $x = \frac{15}{4}$ $x = -4$
 x cannot be a -ve, length.
 so $x = \frac{15}{4}$



$AE = x + 5$
 $AE = \frac{15}{4} + 5$
 $AE = \frac{35}{4}$
 $DE = x + 12$
 $DE = \frac{15}{4} + 12$
 $DE = \frac{63}{4}$

Finding $\hat{A}DE$ using sine rule:

$\frac{\sin A}{a} = \frac{\sin B}{b}$
 $\frac{\sin \hat{A}DE}{AE} = \frac{\sin \hat{D}AE}{DE}$
 $\hat{A}DE = (\frac{\sin(48)}{\frac{63}{4}} \times \frac{35}{4})^{-1}$
 $\hat{A}DE = 24.4^\circ$



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24.4

(Total for Question 23 is 5 marks)

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Turn over for Question 24



24 The diagram shows a solid cone and a solid sphere.

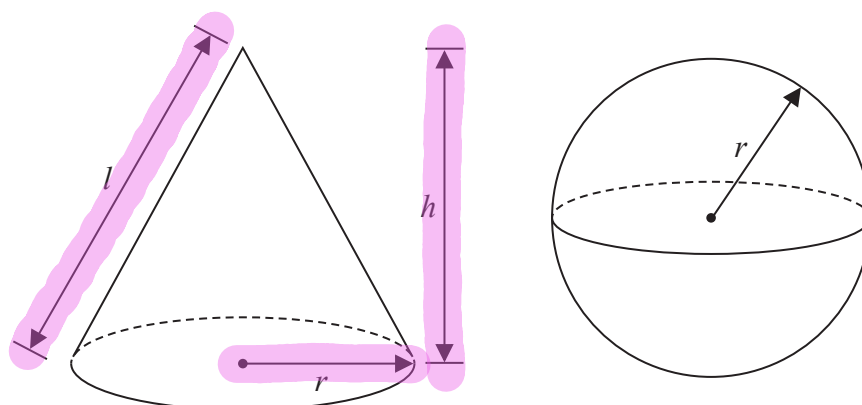


Diagram NOT accurately drawn

The cone has base radius r , slant height l and perpendicular height h
The sphere has radius r

The base radius of the cone is equal to the radius of the sphere.

Given that

$$k \times \text{volume of the cone} = \text{volume of the sphere}$$

show that the **total** surface area of the cone can be written in the form

$$\pi r^2 \left(\frac{k + \sqrt{k^2 + a}}{k} \right)$$

where a is a constant to be found.

$$\text{Volume of cone: } \frac{1}{3} \pi r^2 h$$

$$\text{Volume of sphere: } \frac{4}{3} \pi r^3$$

$$k \times \frac{1}{3} \pi r^2 h = \frac{4}{3} \pi r^3$$

Canceling common terms

$$k \times \frac{1}{3} h = \frac{4}{3} r$$

$$kh = 4r$$

$$h = \frac{4r}{k}$$

h , r and l can form a right angled triangle:



using pythagoras:

$$a^2 + b^2 = c^2$$

$$r^2 + h^2 = l^2$$

$$l^2 = r^2 + \left(\frac{4r}{k}\right)^2$$

$$l = \sqrt{r^2 + \left(\frac{4r}{k}\right)^2}$$

$$l = \sqrt{r^2 + \frac{16r^2}{k^2}}$$

$$l = r \sqrt{1 + \frac{16}{k^2}}$$

Curved surface area of cone: $\pi r l$

$$= \pi \times r \times r \sqrt{1 + \frac{16}{k^2}}$$

$$= \pi r^2 \times \frac{k + \sqrt{k^2 + 16}}{k^2}$$

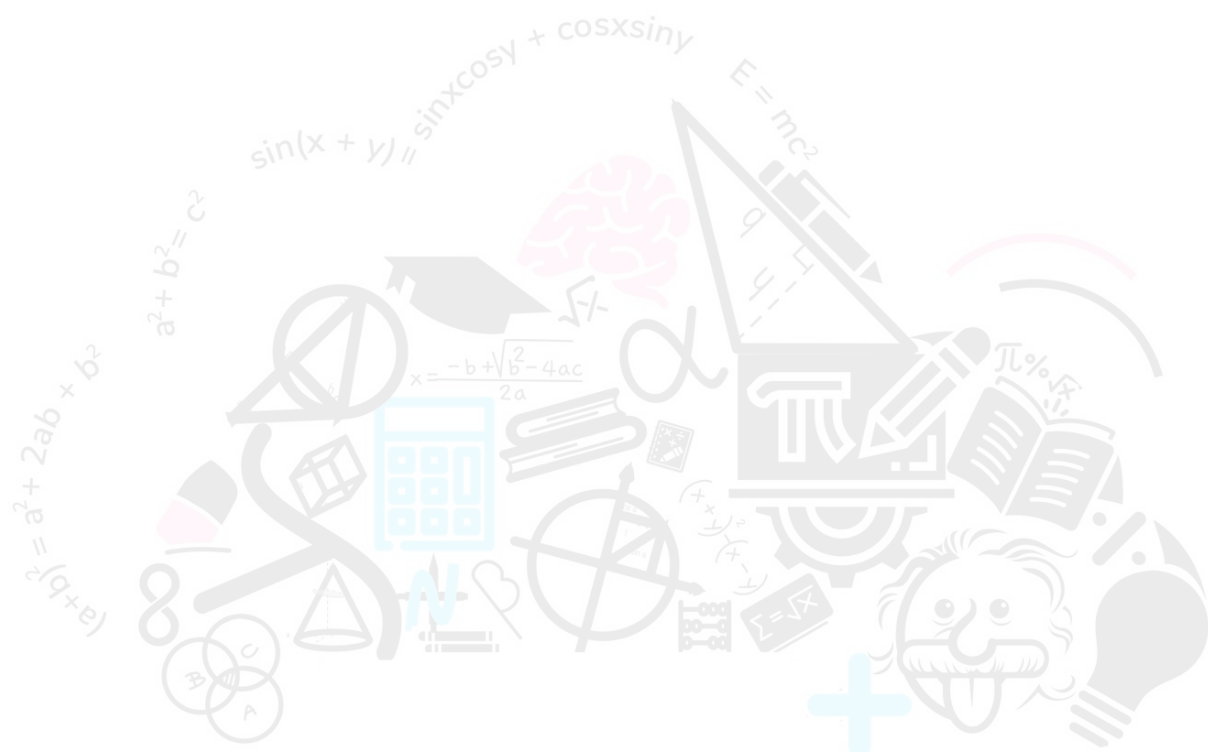
$$= \pi r^2 \left(\frac{k + \sqrt{k^2 + 16}}{k} \right)$$



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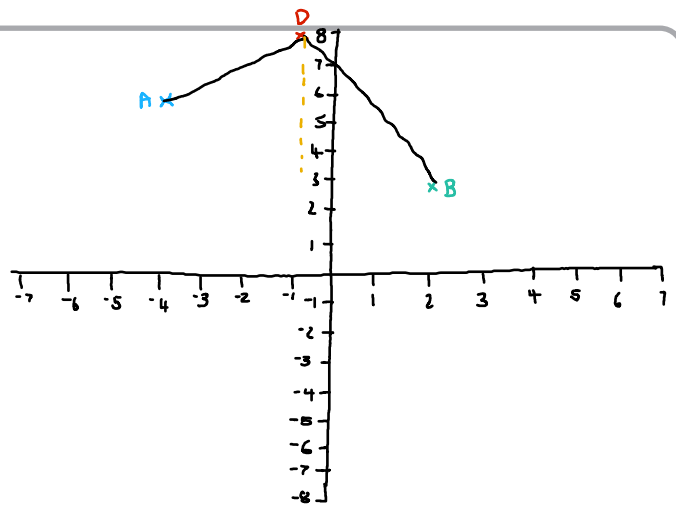
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(Total for Question 24 is 6 marks)

Turn over for Question 25





25 $ABCD$ is a trapezium with AB parallel to DC

A is the point with coordinates $(-4, 6)$

B is the point with coordinates $(2, 3)$

D is the point with coordinates $(-1, 8)$

The trapezium has one line of symmetry.

The line of symmetry intersects CD at the point E

Work out the coordinates of the point E

find midpoint of AB :

$$\text{midpoint: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(-4, 6), (2, 3)$$

$$\text{midpoint } AB: \left(\frac{-4 + 2}{2}, \frac{6 + 3}{2} \right)$$

$$= (-1, 4.5)$$

$$\text{Gradient } AB: \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - 6}{2 - (-4)}$$

$$= -\frac{1}{2}$$

The line of symmetry will be perpendicular to AB , so:

$$m_1 \times m_2 = -1$$

$$-\frac{1}{2} \times m_2 = -1$$

$$m_2 = 2$$

$$\text{midpoint: } (-1, 4.5)$$

$$\text{gradient of midpoint: } \frac{y - 4.5}{x + 1} = 2$$

$$D(-1, 8)$$

$$\text{gradient of } D: \frac{y - 8}{x + 1} = -\frac{1}{2}$$

$$y - 8 = -\frac{1}{2}(x + 1)$$

$$y - 8 = -\frac{1}{2}x - 0.5$$

$$y = -\frac{1}{2}x + 7.5$$

$$\text{plugging into midpoint: } \frac{-\frac{1}{2}x + 7.5 - 4.5}{x + 1} = 2$$

$$-\frac{1}{2}x + 7.5 - 4.5 = 2x + 2$$

$$7.5 - 4.5 = 2.5x + 2$$

$$2.5x = 1$$

$$x = 0.4$$

plug back in to find y :

$$y = -\frac{1}{2}(0.4) + 7.5$$

$$y = 7.3$$

$$= (0.4, 7.3)$$



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(0.4, 7.3)

(Total for Question 25 is 6 marks)

Turn over for Question 26



26 Write

$$\frac{4x^2 - 17x - 15}{2x - 1} \times \frac{2x^2 - 7x + 3}{x^2 - 25} + (29 - 4x)$$

as a single fraction in its simplest form.

Factorising equations:

$$\begin{aligned} &4x^2 - 17x - 15 \\ &x^2 - 17x - 60 \quad \begin{array}{l} \text{as } -20 + 3 = -17 \\ \text{and } -20 \times 3 = 60 \end{array} \\ &= (x - 20)(x + 3) \\ &= (x - 5)(4x + 3) \end{aligned}$$

$$\begin{aligned} &2x^2 - 7x + 3 \\ &x^2 - 7x + 6 \quad \begin{array}{l} \text{as } -6 + (-1) = -7 \\ \text{and } -6 \times (-1) = 6 \end{array} \\ &= (x - 6)(x - 1) \\ &= (x - 3)(2x - 1) \end{aligned}$$

$$\begin{aligned} &x^2 - 25 \\ &\text{difference of 2 squares} \\ &= (x - 5)(x + 5) \end{aligned}$$

Canceling like terms:

$$\frac{(x-5)(4x+3)}{2x-1} \times \frac{(x-3)(2x-1)}{(x-5)(x+5)} + (29-4x)$$

Equation becomes:

$$\begin{aligned} &4x + 3 \times \frac{x-3}{x+5} + 29 - 4x \\ &= \frac{(4x+3)(x-3) + (29-4x)(x+5)}{(x+5)} \end{aligned}$$

Expanding the brackets:

4x	3	29	-4x
x	4x ² 3x	x	29x -4x ²
-3	-12x -9	5	145 -20x
	= 4x ² -9x -9		
	4x ² -9x -9 + (-4x ² +9x +145)		
	x + 5		

Collecting like terms:

$$\begin{aligned} &= \frac{-9 + 145}{x + 5} \\ &= \frac{136}{x + 5} \end{aligned}$$

$$\begin{aligned} &136 \\ &x + 5 \end{aligned}$$

(Total for Question 26 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

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